Altitude Loss in a Split-'S' Maneuver

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Nomenclature

C_L, C_D = lift and drag coefficients
D = drag
g = acceleration of gravity
h = (geometric) altitude
$\bar{h}$ = average between the entrance and exit altitudes
L = lift
n = load factor (measured perpendicular to the flight path)
$\bar{n}$ = representative load factor during the maneuver
p = static pressure
S = wing area
T = thrust
$\bar{T}$ = representative thrust during the maneuver
v = true airspeed
W = weight
α = angle of attack (measured between the zero-lift-line and flight direction)
α_T = angle of the thrust line relative to the zero-lift-line
γ = trajectory angle relative to the horizon
Δh = altitude loss during the maneuver
Δp = static pressure difference between exit and entrance altitudes
η = thrust correction
ρ = air density
$\bar{\rho}$ = air density at $\bar{h}$

Introduction

IN the summer of 1996, an ELBIT-modified MIG21, piloted by the second author, took part in the Paris air show. During the preparation for the show, there was a crucial need to find a minimal safe altitude to initiate a split-'S' maneuver, and, what is more important, to find the sensitivity of this altitude to variations in entrance velocity, weight, weather conditions (temperature and pressure), throttle setting, and angle of attack throughout the maneuver. Toward this end we undertook a small numerical study in which pertinent equations of motion have been solved for several sets of these parameters. The results of this study, described in the next section, inferred that the loss of altitude during the maneuver can be obtained, to a very good accuracy, analytically. Analytical solution for the loss of altitude in a split-'S' is the subject matter of this Note.

Numerical Study

The characteristic time of the maneuver is assumed to be large as compared with the characteristic time of the short-period mode and, therefore, the analysis will be limited to the framework of a point-mass approximation. Pertinent equations of motion are

$$\frac{dh}{d\gamma} = \frac{v^2 \sin\gamma}{g(n - \cos\gamma)} \tag{1}$$

$$\frac{dv}{d\gamma} = \frac{[T\cos(\alpha + \alpha_T) - D - W\sin\gamma]v}{W(n - \cos\gamma)} \tag{2}$$

with

$$n = \frac{L}{W} + \frac{T\sin(\alpha + \alpha_T)}{W} \tag{3}$$

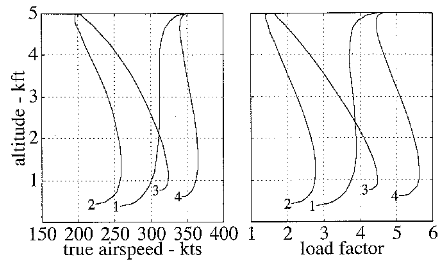

Fig. 1 Altitude histories of the true airspeed and load factor (defined normal to the flight path) in four split-'S' simulations from 5000 ft in standard atmosphere. The throttle setting is idle (10% maximum dry thrust) in runs 1 and 2 and maximum AB in runs 3 and 4; aircraft weight is 15,700 lb and life coefficient is 0.8.

To obtain the loss of altitude in a split-'S', these equations need to be integrated from π to 2π (corresponding to the beginning and end of the maneuver, respectively). The process of numerical integration is straightforward. The difficulty is in modeling the drag and thrust as functions of flight conditions. In the construction of both models we have closely followed chapters 12 and 13 of Rymer.[1] Some of the pertinent empirical coefficients (as skin-friction coefficient, leading-edge suction, and inlet efficiency) have been adjusted to fit the specific power measured in full-throttle maneuvers during the rehearsal flight. With the adjusted models, the specific power was recovered to within 50 ft/s.

Given the adjusted models, a number of split-'S' maneuvers has been simulated for different entrance velocities, weights, throttle settings, and lift coefficients. The results of four of those simulations (with the throttle and lift coefficient set constant throughout the maneuver) are summarized in Fig. 1. It appears that the loss of altitude is practically independent on the entrance velocity and weakly dependent on the throttle setting (it decreases with an increase in thrust), despite significant differences in both velocity and load factor during the respective maneuvers. Hence, it seems plausible that the loss of altitude may be estimated in the framework of a simple analytical analysis. Such an analysis is presented next.

Analysis

Using the standard definition of the lift coefficient, it immediately follows from Eq. (3) that

$$v^2 = \frac{2nW - 2T\sin(\alpha + \alpha_T)}{\rho S C_L} \tag{4}$$

and, therefore

$$\rho g \frac{dh}{d\gamma} = \frac{2W}{SC_L} \frac{n\sin\gamma}{(n - \cos\gamma)} \left[1 - \frac{T\sin(\alpha + \alpha_T)}{nW}\right] \tag{5}$$

by Eq. (1). The expression on the left-hand side of Eq. (5) will be recognized as the derivative $dp/d\gamma$ of the atmospheric pres-

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sure (taken with the negative sign); hence, an integration of Eq. (5) between π and 2π readily yields the pressure rise

$$\Delta p = -\int_{\pi}^{2\pi} \frac{2W}{SC_L} \frac{n \sin \gamma}{(n - \cos \gamma)} \left[1 - \frac{T \sin(\alpha + \alpha_T)}{nW}\right] d\gamma \qquad (6)$$

between the exit and entrance altitudes. Given the pressure altitude (and, hence, the pressure) at the beginning of the maneuver, Δp straightforwardly yields the pressure altitude at the end of it. However, in low-altitude aerobatics, a pilot is usually concerned with the loss of geometric, rather than pressure, altitude. The former, which will be denoted Δh, can be estimated from Δp as follows.

Let $\bar{h}$ be the arithmetic average between the entrance and exit altitudes. Expanding the pressure into a Taylor series about $\bar{h}$, one has

$$\Delta p = -\left(\frac{dp}{dh}\right)_{h=\bar{h}} \Delta h - \frac{1}{24}\left(\frac{d^3p}{dh^3}\right)_{h=\bar{h}} \Delta h^3 - \cdots \qquad (7)$$

But $(dp/dh)_{h=\bar{h}} = -\bar{\rho}g$, where $\bar{\rho}$ is the air density at $\bar{h}$. Thus, in the leading order

$$\Delta h = -\frac{2W}{\bar{\rho}gS} \int_{\pi}^{2\pi} \frac{n \sin \gamma}{C_L(n - \cos \gamma)} \left[1 - \frac{T \sin(\alpha + \alpha_T)}{nW}\right] d\gamma \qquad (8)$$

One can easily verify that for a loss of 5000 ft in standard atmosphere, the error introduced in Eq. (8) is less than 0.1%.

A low-altitude minimum-altitude-loss split-'S' is commonly executed on the verge of buffeting; and, therefore, Eq. (8) can be significantly simplified by assuming C_L constant:

$$\Delta h = -\frac{2W}{\bar{\rho}gSC_L} \int_{\pi}^{2\pi} \frac{n \sin \gamma}{n - \cos \gamma} \left[1 - \frac{T \sin(\alpha + \alpha_T)}{nW}\right] d\gamma \qquad (9)$$

Now, let us further simplify the problem by assuming that the normal-to-the-flight-path thrust component is negligible compared with aerodynamic lift; namely that

$$T \sin(\alpha + \alpha_T) \ll nW \qquad (10)$$

the consequences of this assumption will be discussed in the next section. With Eq. (10), Eq. (9) becomes

$$\Delta h = -\frac{2W}{\bar{\rho}gSC_L} \int_{\pi}^{2\pi} \frac{n \sin \gamma}{n - \cos \gamma} d\gamma \qquad (11)$$

The load factor n undergoes considerable variations during the maneuver (Fig. 1). Nonetheless, we suggest that the loss of altitude is insensitive to a specific value of n. To prove this suggestion, consider the integral

$$I(a) = -\int_{\pi}^{2\pi} \frac{a \sin \gamma}{a - \cos \gamma} d\gamma \qquad (12)$$

where a is a constant. Obviously

$$I(a) = a\, \ell n \frac{a + 1}{a - 1} = 2 + \frac{2}{3a^2} + \frac{2}{5a^4} + \cdots \qquad (13)$$

It immediately follows from Eq. (13) that I is, practically, a constant, it varies only by 10% on the interval $(2, \infty)$, and by 2% on the interval $(4, \infty)$ (see Fig. 2). Accordingly

$$\Delta h = \frac{2WI(\bar{n})}{\bar{\rho}gSC_L} \qquad (14)$$

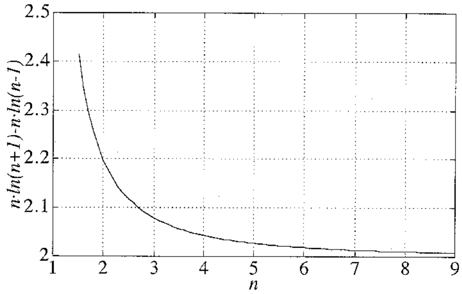

Fig. 2 Function $I(n)$.

where $\bar{n}$ is a certain representative load factor during the maneuver. This load factor corresponds to neither the beginning nor the end of the maneuver, at these instances the integrand in Eq. (11) is vanishingly small. Because, under most circumstances, the load factor in midmaneuver will be greater than 2, the altitude loss can, most likely, be bounded by

$$\frac{4W}{\bar{\rho}gSC_L} < \Delta h \leq \frac{4.4W}{\bar{\rho}gSC_L} \qquad (15)$$

[to obtain Eq. (15), substitute $2 \leq \bar{n} < \infty$ in Eq. (14)]. Thus, air density, lift coefficient, and wing loading are the most important factors that determine the loss of altitude in a split-'S'; this loss is insensitive to both the throttle setting, as long as it satisfies Eq. (10) and the initial airspeed, as long as it allows C_L to remain constant throughout the maneuver.

With $W = 15{,}700$ lb, $S = 248$ ft^2, $C_L = 0.8$, and $\bar{\rho} = 0.0702$ lb/ft^3 (corresponding to the altitude of 2800 ft in standard atmosphere), Eq. (15) bounds Δh to be no greater than 4960 ft and no less then 4510 ft. Numerical simulations for runs 1 and 2 (executed with idle thrust) yield 4610 and 4560 ft, respectively.

Thrust Effects

An increase in thrust reduces the loss of altitude in two ways. First, it increases the airspeed and, hence (for a given lift coefficient), the load factor. The results of the preceding section imply that this effect is most likely small. Second, there exists a normal-to-the-flight-path thrust component [the rightmost term in Eq. (3)], which increases the load factor directly. This effect is addressed next.

By analogy with Eq. (14), Eq. (9) can be written in the following form:

$$\Delta h \approx [2WI(\bar{n})/(\bar{\rho}C_L gS)](1 - \eta) \qquad (16)$$

where

$$\eta = (\bar{T}/W)[\sin(\alpha + \alpha_T)/\bar{n}] \qquad (17)$$

is the pertinent thrust correction. It immediately follows from Eq. (17) that this correction is highly sensitive to the specific value of $\bar{n}$. Because the latter is a priori unknown, Eq. (16) can only be used to provide a very rough estimate. It is clear, however, that η will be small for either high g or partial-throttle/high-altitude maneuvers (where the thrust will be only a fraction of the maximum sea-level thrust). In those cases Eq. (14) or (15) can be conveniently used to estimate the loss of altitude.

With $\bar{T} \approx 10{,}300$ lb (approximate maximum installed thrust at 300 KTAS and 3000 ft), $W = 15{,}700$ lb, $\alpha \approx 13$ deg (corresponding to $C_L = 0.8$ with partial flaps) and $\alpha_T = 0$, Eq. (17) yields η between 0.05 and 0.1 for $\bar{n}$ between 3 and 1.5, and η between 0.03 and 0.05 for $\bar{n}$ between 4.5 and 3. That is, the loss of altitude in run 3 (max AB, 200 KTAS) should be, roughly, 450–250 ft less than in run 2 (idle, same airspeed), whereas the loss of altitude in run 4 (max AB, 350 KTAS)

Table 1 Summary for two split-'S' maneuvers

Parameter	No. 1	No. 2	Units
Entrance altitude	8880	6850	ft
Entrance velocity	320	340	KTAS
$\bar{\rho}$	0.0628	0.0663	lb/ft^3
W	15,100	14,800	lb
Average C_L	0.80	0.81	——
Average α	13.0	13.3	deg
Average n	4.5	4.6	——
Δh measured	4890	4340	ft
Δh estimated by Eq. (14)	5010	4580	ft
Δh estimated by Eq. (15)	>4930	>4510	ft
Δh estimated by Eq. (16)	4860	4430	ft

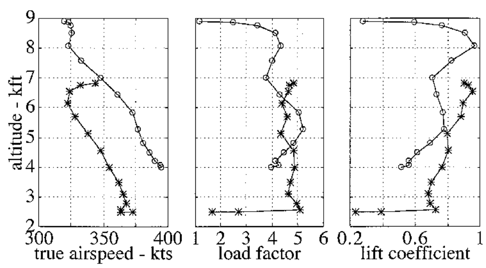

Fig. 3 Altitude histories of the true airspeed, load factor, and lift coefficient in two full-throttle split-'S' maneuvers.

should be, roughly, 250–150 ft less than in run 1 (idle, same airspeed). These estimates accord the numerical simulation, altitude loss in run 3 is about 300 ft less than in run 2, whereas the loss in run 4 is about 200 ft less than in run 1.

Flight Tests

Altitude histories of two sample full-throttle maneuvers are shown in Fig. 3; pertinent data are summarized in Table 1. Aircraft acceleration, velocity, and position have been taken from its navigation system. The lift coefficient was estimated from the acceleration, angle of attack, and either model thrust or model drag; the thrust- and drag-based estimates have agreed to within 1%. Midmaneuver density $\bar{\rho}$ has been computed using the actual temperature and pressure on the test day (that was approximately ISA+3). Average lift coefficient, angle of attack, load factor, and aircraft weight have been computed on the part of the maneuver where the flight-path angle γ was between $7/6\pi$ and $11/6\pi$ (the mid-two-thirds of the maneuver). Based on these averaged values, Eq. (14) overestimates the actual altitude losses of the two runs by 120 and 240 ft. As compared with the altitude loss of about 4500 ft, the accuracy of this approximation is about 5%. Thrust correction turns out to be roughly 150 ft by Eq. (17), whereas Eq. (16) underestimates the altitude loss in the first run by 30 ft and overestimates the loss in the other by 90 ft.

Summary

As compared with both flight tests and numerical simulations, Eq. (15) seems to predict the loss of altitude in a split-'S' with reasonable accuracy. This loss depends, basically, on the wing loading, air density, and the lift coefficient; it is insensitive to variations in the entrance velocity (as long as the lift coefficient remains unlimited by the structural limit), and the throttle setting. Still, an altitude loss in an open-throttle split-'S' will typically be less than in a comparable maneuver executed with throttle on idle.

Reference

[1]Rymer, D., *Aircraft Design. A Conceptual Approach*, AIAA Educational Series, AIAA, Washington, DC, 1991, pp. 255–321.

Base Drag Reduction Caused by Riblets on a GAW(2) Airfoil

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Nomenclature

C_{DB} = base drag coefficient, $[-C_{pb}(t/c)]$
C_{DT} = total drag coefficient
C_p = pressure coefficient, $(p - p_\infty)/q_\infty$
C_{pb} = base pressure coefficient, $(p_b - p_\infty)/q_\infty$
c = airfoil chord
h = riblet height
h^+ = hu_*/ν
p = local static pressure
p_∞ = freestream static pressure
q_∞ = freestream dynamic pressure
t = trailing-edge thickness
u_* = friction velocity
x = distance along the chord
y = distance normal to tunnel axis
α = angle of attack
ΔC_D = $(C_{D\text{riblet}} - C_{D\text{noriblet}})$
ν = kinematic viscosity

Introduction

AMONG various methods explored for turbulent drag reduction on aerodynamic surfaces, riblets have been the most promising.[1] As much as 4–8% of viscous drag reduction has been reported for simple two-dimensional configurations. Plastic sheets with symmetric v-grooves (manufactured by the 3M Co.) have been employed widely in research. Assessment of viscous drag reduction on two-dimensional airfoils, both at low and transonic speeds, has been reported as well.[2–6] Excellent reviews on the subject covering aspects of drag reduction and flow structure are contained in Refs. 1 and 7.

There have been very few attempts exploring the use of riblets in separated flows, either from the point of view of drag reduction or separation control.[8,9] Recently, Krishnan et al.[8] showed that riblets actually increase the base drag (about 8%) on a long axisymmetric body with a blunt base at low speeds; the base diameter was about four times the boundary-layer thickness ahead of the base corner. They used 3M riblet sheets and systematically studied the effect of h^+ on base pressure. They also speculated that, while riblets caused an increase in the base drag for a large-scale separated flow (like on the axisymmetric blunt base[8]), the effect could be favorable on an airfoil with a blunt trailing edge, which is a case of a small-scale separated flow.

The present investigation was undertaken specifically to assess the effect of 3M riblets on the base pressure of an airfoil with a blunt trailing edge. Experiments were made at low speeds on a 13.6% thick GAW(2) airfoil model, which has a trailing-edge thickness ratio of 0.5%. The results show very clearly that the base drag reduction of an engineering value can be achieved for the optimized riblet geometry.

Experiments

Facility and Model

The experiments were conducted in a 300 × 1500 mm boundary-layer tunnel. The GAW(2) airfoil model, with a c of

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